Apportioning Foreign Exchange Risk Through the Use of Third Currencies: Some Questions on Efficiency

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Introduction

In a world characterized by growing uncertainties in foreign exchange markets, currency exchange risk is of obvious concern in transactions involving residents of different countries. The existence of currency borders and international exchange risks may represent not simply a burden on contracting parties but may even prevent otherwise profitable transactions. This would be the case if either party viewed risk as not commensurate with expected returns.

Careful apportioning between the contracting parties of risks attributable to exchange rate fluctuations may make feasible a transaction that would be blocked altogether if either party had to bear the full brunt of potential losses. With the advent of flexible exchange rates, contracts in third currencies, including the recently devised composite units of account, seem to be used increasingly to apportion risk. These approaches to balancing the exchange risks have met with varying degrees of success that the literature is only just beginning to explore [2, 3]. The focus of recent articles has been descriptive, with particular emphasis on international financing alternatives available to the firm. While debt contracts denominated in increasingly complex currency baskets are being devised, the question whether it is even reasonable and desirable to contemplate third currency instruments has not yet been analyzed systematically. In this paper we wish to explore and question the efficiency of contracting in third currencies, including units, in order to apportion exchange risk. If we consider the denomination of debt issues in European capital markets over the past two or three years, the question is clearly relevant.

The results obtained will depend on the measure of risk applied. Where exchange risk is viewed as the variations in nominal returns (or costs) which accrue to the transacting parties, as measured in their respective domestic currencies, we show that two contracting parties accounting in different currencies can always achieve a better sharing of risk by confining themselves to a mix of their currencies. There is less risk for each party in such cases than in using any third currency as the basis for a contract. Only where risk is perceived as the variation of real returns, based on some reference basket of goods, may the resort to third currencies or units provide an efficient apportioning of risk that could not be obtained by other measures.

While we recognize that, by focusing solely on exchange rates, our model is limited and analyzes only a partial equilibrium, it nevertheless serves to draw attention to some important issues in the management of foreign exchange risks.

The Basic Model

Consider a borrower and lender dealing across currency borders. For clarity of exposition, we assume a simple one-period model; borrowing takes place at the beginning of the period, and principal is repaid with interest at the end of the period. Additionally, the risk of international loan transactions is equated with the variance of returns. While this may not be in keeping with some of the conceptual approaches to finance that involve portfolio theory, it is a completely reasonable and frequently used simplification at the level of the firm where financial officers may not be permitted the luxury of taking a portfolio approach [4]. It might be borne in mind that a portfolio approach may not be appropriate where entrepreneurs regularly risk a major portion of their total wealth. This is relevant in our context since market participants in foreign currency transactions often risk very significant sums. Focusing on foreign exchange risk as defined above may also be justified by the embarrassment of managers at having to report foreign exchange losses, leading to an aversion to foreign exchange risk *per se*.

The existence of forward exchange markets provides evidence of aversion to foreign exchange risk on the part of a substantial segment of those who make transactions in international currency markets. The lack of such markets for distant maturities, however, may leave the long-term borrower or lender exposed to the risk of currency fluctuations, and other arrangements may have to be found for minimizing this risk. We note that while our model is cast in the context of loan transactions across currency boundaries, the results obtained are equally relevant to payments for any other trade transaction.

Assume that a fraction X_i ($i = 0, 1, \ldots, n$) of the loan is denominated in the currency of country i and that the remaining fraction ($\sum_{i=0}^{n}$) is denominated in the reference currency in terms of which exchange rates are defined. The borrower's country is country 0, the lender's country 1.

The exchange rate for currency i ($i = 0, 1, \ldots, n$) at the start of the period is denoted by e_{0i}, and the corresponding exchange rate at the end of the period by e_{1i}. All other exchange rates may be derived from these basic rates: thus e_{0i}/e_{0j} is the number of units of i per unit of currency j.

The following notation will also be used: R_L is the anticipated rate of return to the lender, computed in the lender's currency; R_B the anticipated rate of cost to the borrower, computed in the borrower's currency; and k the rate of interest on the loan.

In order to focus precisely on the issue of exchange risk, it is assumed that the nominal interest rate in all currencies is constant at k. This involves no loss of generality since the coupon rate on the loan may be adjusted after determining the optimal currency denomination.

The gross rate of return to the lender $(1 + R_L)$ measured in his own currency may be expressed as:

$$(1 + R_L) =$$

$$(1 + k) \left[\sum_{i=0}^{n} X_i \frac{e_{0i} e_{11}}{e_{01} e_{1i}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{e_{11}}{e_{01}} \right]. \qquad (1)$$

We shall, however, find it convenient to work with a minor modification of Equation (1), namely

$$Y_L = \frac{1+R_L}{1+k} = \sum_{i=0}^{n} X_i \frac{e_{0i} e_{11}}{e_{01} e_{1i}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{e_{11}}{e_{01}}. \qquad (1a)$$

Similarly, $Y_B = (1 + R_B)/(1 + k)$, where $(1 + R_B)$ is the gross cost to the borrower measured in his currency, is given by

$$Y_B = \frac{1+R_B}{1+k} = \sum_{i=0}^{n} X_i \frac{e_{0i} e_{10}}{e_{00} e_{1i}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{e_{10}}{e_{00}}. \qquad (2)$$

An efficient currency denomination scheme (X_0, $X_1, \ldots X_n$) can be thought of as one which minimizes the weighted sum

$$V = \sigma^2(Y_L) + \lambda\, \sigma^2(Y_B). \qquad (3)$$

As is derived mathematically in the appendix, a minimum for V implies the following currency proportions:

$$X_0 = \frac{\lambda}{1+\lambda}$$

$$X_1 = \frac{1}{1+\lambda}$$

$$X_i = 0\,(i \neq 0, 1).$$

Thus the optimal currency denomination scheme requires that the loan be denominated only in the lender's and borrower's currencies. Given our framework, it would never be optimal to denominate any part of the transaction in a third currency.

Use of Third Currencies where Returns and Costs are Measured in Real Terms

Clearly, the above conclusion contradicts current practice. Many international transactions are denominated in third currencies. Also, when various mixed currency instruments and units have been introduced to the market, many have found ready acceptance. Two notions may explain this apparent discrepancy between theory and practice. One possibility would be to dismiss the practice of resorting to third currencies as irrational conduct on the part of financial managers and investors. Perhaps they are unaware that the positions of both a lender and borrower can be improved if the transaction is based on a mix of their two domestic currencies. Since the result derived above is not immediately obvious, we suspect that such a lack of understanding may indeed have caused the use of a third currency when its use was actually disadvantageous. However, even if individual cases of suboptimal behavior appear plausible, it would be unreasonable to view this as the major cause for the discrepancy. Alternatively, we can modify the assumptions of the above model to search for a reconciliation between theory and practice.

It is well known that borrowing in a third currency can serve to reduce variations in reported income as caused by exchange rate changes if, for example, the borrowing entity has assets or income denominated in such currency. More generally, the real returns or costs that are of concern to a lender or borrower in a particular transaction may not be measured properly by reference to their respective domestic currencies, but rather by reference to some basket of goods. Such baskets of goods need not coincide with the mix of items making up various widely used price indices, but they are likely to reflect the more significant incomes and payments and assets and liabilities of the contracting parties. Where general price level changes or inflation are of concern, and the reference basket of goods does coincide with some general price index, resort to a third currency that is deemed to be strong or stable may serve as an alternative to indexation. Currencies (or units) that have been popular from time to time as the basis of international transactions include the United States dollar, the German mark, the Swiss franc, and more recently even "baskets" such as the SDR [1, 5]. At the time, these currencies were generally viewed as being strong, meaning that their purchasing power in terms of some sought-after basket of goods is less likely to erode over time than that of other monetary units.

For the sake of simplicity, we will only indicate by examples why and how our earlier conclusions are modified in this more general context. (The mathematical derivation of general results may be found in the appendix.) We start by assuming identical reference baskets for two contracting parties. The value of each currency can then be defined in relation to the reference basket of goods which we may assume to take the place of the reference currency used in the previous section. R_L^*, the real return to the lender, can be expressed as

$$(1 + R_L^*) = (1 + R_L)e_{01}/e_{11}. \qquad (4)$$

Similarly, for the borrower the real cost becomes

$$(1 + R_B^*) = (1 + R_B)e_{00}/e_{10}. \qquad (5)$$

If the loan takes place in a third currency j, it is apparent that

$$(1 + R_L^*) = (1 + k)\frac{e_{11}e_{0j}}{e_{1j}e_{01}} \times \frac{e_{01}}{e_{11}}$$

$$= (1 + k)\frac{e_{0j}}{e_{1j}} \qquad (6)$$

and

$$(1 + R_B^*) = (1 + k)\frac{e_{0j}e_{10}}{e_{00}e_{1j}} \times \frac{e_{00}}{e_{10}}$$

$$= (1 + k)\frac{e_{0j}}{e_{1j}}. \qquad (7)$$

Thus, the real returns/costs to the lender/borrower are only determined by the change in value of the contract currency j *vis-à-vis* the reference.

If the loan is denominated in a mix of the borrower's and the lender's currencies, with p the proportion of the loan denominated in the borrower's currency, we obtain

$$(1 + R_L^*) =$$

$$\left[(1-p)(1+k) + p(1+k)\frac{e_{11}e_{00}}{e_{10}e_{01}} \right] \frac{e_{01}}{e_{11}}$$

$$= (1+k)\left[(1-p)\frac{e_{01}}{e_{11}} + p\,\frac{e_{00}}{e_{10}}\right] \qquad (8)$$

and

$$(1 + R_B^*) = (1+k)\left[(1-p)\frac{e_{01}}{e_{11}} + p\,\frac{e_{00}}{e_{10}}\right]. \qquad (9)$$

This time, therefore, real returns and costs are determined by changes in value of both lender's and borrower's currencies *vis-à-vis* the reference.

It is clear from the above that one can now find circumstances where use of a third currency will lead to even smaller risks for both borrower and lender than use of a mix of their respective two currencies. One need only consider the extreme case, where the value of a third currency is perfectly correlated with the value of the predetermined basket of goods, so that var $(1/e_{1J})$ becomes zero, with the perceived variances of e_{11} and e_{10} being positive. Contrary to the case where we were simply concerned with nominal returns, even where the currencies of lender and borrower are strongly correlated, it now may pay to denominate a transaction in a third currency that is less correlated with either the borrower's or the lender's currency but that exhibits price stability. While this will cause the nominal returns and costs R_L and R_B to be subject to increased uncertainties, the variance of the real returns and costs R_L^* and R^*_B can be reduced.

By way of example, consider the extreme case where both the borrower and the lender account in the same currency, so that $e_{00} = e_{01} = e_0$, and $e_{10} = e_{11} = e_1$. From Equations (6) and (7), we have

$$\text{var}\left(\frac{1+R}{1+k}\right) = e_{0J}^2 \text{ var}(1/e_{1J}),$$

while Equations (8) and (9) yield

$$\text{var}\left(\frac{1+R}{1+k}\right) = e_0^2 \text{ var}(1/e_1).$$

Thus, so long as var $(1/e_{1J}) <$ var $(1/e_1)$, that is, as long as the reference basket of goods is deemed to exhibit greater price stability with regard to a third currency than with regard to the domestic currency, real return and costs can be stabilized by denominating the transaction in the third currency. Even a mix of various currencies can become the optimal reference unit if the relevant basket of goods exhibits price stability in terms of the mix. For example, where various goods in the basket are likely to be stable with regard to various currencies, a mix of such currencies may conceivably out-perform any individual currency. A derivation of the optimal currency mix under general conditions is provided in the appendix.

Clearly, one could achieve even better results by using the preferred reference baskets of goods as a basis for the transaction. Should the two contracting parties agree on the appropriate basket of goods to be used as a reference, the risks in real returns and costs could be completely eliminated. Where the appropriate reference basket varies between borrower and lender, following arguments set out in the first section of this paper, conceptually the most efficient approach would be to base the transaction on a weighted average of the two baskets. Use of any currency scheme is bound to be less efficient.

Currencies as a basis for a transaction may, however, clearly be more convenient to use than baskets of goods. Where one can find currency mixes that can be termed stable in terms of their purchasing power with regard to certain reference baskets, their use as a basis for international transactions is reasonable and consistent with our results. We have seen that this will be common where the transacting entities have international operations involving receipts and disbursements denominated in various currencies, and where such firms may be concerned about shielding their reported earnings figures from fluctuations in foreign exchange rates. Resort to third currencies as an alternative to general indexation may have been prevalent in the past. Today, however, with world-wide inflation and growing instability of even those currencies which were traditionally viewed as strong, the use of third currencies in this context becomes increasingly questionable. Where unanticipated inflation and the risks of real costs and returns are of concern, indexation based on some representative basket of goods, or on weighted averages of such baskets where there are differences for the transacting parties, will have to be considered seriously.

Finally, we note that various market imperfections, such as particularly high transactions costs or the prospect of currency controls may cause transacting parties to prefer some currencies over others. On the other hand, a transacting party may be willing to accept denomination in a less-than-optimal currency if the interest rate differential differs from the expected rate of change in the exchange rate by more than the required return for bearing the added exchange risk.

Conclusion

This paper, a natural follow-up to the excellent descriptive piece by Robichek and Eaker [3], explores the question of how international loans should be denominated. Can one, in other words, justify the use of SDRs, EURCOs, B-Bonds, EEC-EUAs, or ARCRUs if one is concerned about minimizing the total risks to be borne by both borrower and lender? One conclusion is that, where returns and costs are measured in the respective domestic currencies, debt and other contracts should only be denominated in the currencies of the transacting parties. It is never optimal to denominate any part of a transaction in a third currency or a mix of third currencies. Examples where the reference of domestic currencies may be valid include foreign borrowing by local governments (where future costs have to be met from local tax revenues), or lending by certain financial institutions (where the liabilities may be in terms of the domestic currency).

Once we recognize that in a more general context, however, real returns and costs may be measured in reference units other than the domestic currencies of the transacting parties, the choice of third currencies, including units of various types, may make practical sense; smaller risks in real returns and costs may be possible for both borrower and lender than would be the case otherwise. Therefore, we conclude that third currencies including units will not aid in apportioning foreign exchange risk, but that they may serve as a convenient way of stabilizing real returns and costs as perceived by the transacting parties. We have shown optimal currency schemes under general conditions for each of these alternatives. Empirical work could usefully be undertaken as a subsequent stage in this area of research.

It should be noted that, while our focus has been the denomination of debt instruments, our analysis should also provide insights into other areas of financial management. To pick just one possible example, longer-term international contracts involving natural resources such as coal are quite common today. Pricing is one important aspect of such agreements, and it is clear that the choice of currency could be critical. Certainly, what we have explored about optimal currency denominations is also relevant in this more general context of foreign transactions.

Appendix

We first derive the optimal mix of currencies where both transacting parties are concerned about the nominal returns/cost as measured in their respective domestic currencies. Using as a starting point Equations (1a) and (2) of the paper, and letting $e_{1i} = e_{0i} + \Delta e_i$ and $e_{1j} = e_{0j} + \Delta e_j$, then

$$\frac{e_{0i}e_{1j}}{e_{0j}e_{1i}} = \frac{1 + \Delta e_j/e_{0j}}{1 + \Delta e_i/e_{0i}} \approx 1 + d_j - d_i \qquad \text{(A-1)}$$

where $d_i = \Delta e_i/e_{0i}$, and terms of higher order than the first have been dropped. Similarly,

$$\frac{e_{1j}}{e_{0j}} = 1 + d_j. \qquad \text{(A-2)}$$

Then substituting Equations (A-1) and (A-2) in (1a) and (2) we obtain

$$Y_L = \sum_{i=0}^{n} X_i(1 + d_l - d_i) + (1 - \sum_{i=0}^{n} X_i)(1 + d_l)$$

$$Y_L = 1 - \sum_{i=0}^{n} X_i d_i + d_l \qquad \text{(A-3)}$$

$$Y_B = 1 - \sum_{i=0}^{n} X_i d_i + d_0. \qquad \text{(A-4)}$$

Equations (A-3) and (A-4) decompose the exchange loss or gain into two components. Thus, for the borrower, d_0 represents the exchange loss if the whole loan were denominated in the reference currency; $\sum_{i=0}^{n} X_i d_i$ then represents the exchange gain in relation to the reference currency.

From Equations (A-3) and (A-4) we may write the variances of Y_L and Y_B as

$$\sigma^2(Y_L) = \sum_{i=0}^{n}\sum_{j=0}^{n} X_i X_j \sigma_{ij} - 2\sum_{j=0}^{n} X_j \sigma_{1j} + \sigma_{11} \qquad \text{(A-5)}$$

$$\sigma^2(Y_B) = \sum_{i=0}^{n}\sum_{j=0}^{n} X_i X_j \sigma_{ij} - 2\sum_{j=0}^{n} X_j \sigma_{0j} + \sigma_{00} \qquad \text{(A-6)}$$

where $\sigma_{ij} = \text{cov}(d_i, d_j)$.

As noted in the paper, we want to minimize the weighted sum

$$V = \sigma^2(Y_L) + \lambda\sigma^2(Y_B). \qquad \text{(A-7)}$$

The first order conditions for a minimum in Equation (A-7) are

$$\frac{dV}{dX_i} = 2(1 + \lambda)\sum_{j=0}^{n} X_j \sigma_{ij} - 2\sigma_{i1} - 2\lambda_{0i} = 0 \qquad \text{(A-8)}$$

$$(i=0, 1, \ldots, n)$$

Equation (A-8) may be written in matrix notation as

$$(1 + \lambda)\, \Omega X = (\Omega_1 + \lambda \Omega_0) \qquad \text{(A-9)}$$

where X is the (n + 1) vector of optimal currency proportions, Ω is the (n + 1, n + 1) variance-covariance matrix with first column Ω_0 and second column Ω_1.

Solving Equation (A-9) for X,

$$X = \frac{1}{1+\lambda}\, \Omega^{-1} (\Omega_1 + \lambda \Omega_0). \qquad \text{(A-10)}$$

By the usual properties of a matrix inverse $\Omega^{-1}\Omega_0 = (1, 0, 0, \ldots 0)'$ and $\Omega^{-1}\Omega_1 = (0, 1, 0, 0, \ldots, 0)'$.

Thus Equation (A-10) implies the result stated in the paper, namely

$$X_0 = \frac{\lambda}{1+\lambda}$$

$$X_1 = \frac{1}{1+\lambda}$$

$$X_i = 0 \text{ for } i \neq 0, 1.$$

We now address the more general case. Costs and returns are measured in real terms with regard to some reference basket of goods, where this basket of goods need not be the same for both transacting parties. We use the symbol P to denote the exchange rate between the reference basket and the reference currency, retaining the previous notation on subscripts, so that, for example, P_{11} denotes the number of units of the lender's reference basket to purchase one unit of the reference currency at the end of period 1. It is clear that the real returns to the lender measured in units of his reference basket now become

$$(1 + R_L^*) =$$

$$(1 + k) \left[\sum_{i=0}^{n} X_i \frac{e_{0i} P_{11}}{P_{01} e_{1i}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{P_{11}}{P_{01}} \right] \qquad \text{(A-11)}$$

with

$$Y_L^* = \frac{1+R_L^*}{1+k} = \sum_{i=0}^{n} X_i \frac{e_{0i} P_{11}}{P_{01} e_{1i}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{P_{11}}{P_{01}}. \qquad \text{(A-11a)}$$

Similarly, for the borrower we obtain

$$Y_B^* = \frac{1+R_B^*}{1+k} = \sum_{i=0}^{n} X_i \frac{e_{0i} P_{10}}{e_{1i} P_{00}} + \left(1 - \sum_{i=0}^{n} X_i\right) \frac{P_{10}}{P_{00}}. \qquad \text{(A-12)}$$

Following the same reasoning as above, we can write

$$Y_L^* = 1 - \sum_{i=0}^{n} X_i d_i + d_{P1}$$

$$Y_B^* = 1 - \sum_{i=0}^{n} X_i d_i + d_{P0}$$

where d_P are the changes in the exchange rate of the reference baskets *vis-à-vis* the reference currency. Forming again

$$V = \sigma^2(Y_L^*) + \lambda\, \sigma^2(Y_B^*)$$

from the necessary conditions for a minimum, we now have

$$(1 + \lambda)\, \Omega X = \pi_1 + \lambda \pi_0 \qquad \text{(A-13)}$$

where π_1 and π_0 stand for the column vectors of covariances between the d_i's and d_{P1} and d_{P0} respectively. With

$$X = \frac{1}{1+\lambda}\, \Omega^{-1} (\pi_1 + \lambda\, \pi_0) \qquad \text{(A-14)}$$

it is easy to see that the optimal currency scheme may now involve a mix of any of the n + 1 currencies, with the optimal mix determined as per Equation (A-14).

For the special case where the reference baskets are identical for borrower and lender, with $\pi_1 = \pi_0 = \pi$, we obtain

$$X = \Omega^{-1} \pi.$$

Where the lender's and borrower's reference baskets are simply their respective currencies, with $\pi_1 = \Omega_1$ and $\pi_0 = \Omega_0$, we confirm our earlier result, with

$$X = \frac{1}{1+\lambda}\, \Omega^{-1} (\Omega_1 + \lambda\, \Omega_0).$$

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